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Engineering Mathematics - III
(124111 / 214111)

P. Pages : 3

Time : Three Hours **Max. Marks : 80**

Instructions to Candidates :
1. Do not write anything on question paper except Seat No.
2. Answer sheet should be written with blue ink only. Graph or diagram should be drawn with the same pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. Attempt **any two** sub-questions from each unit.
5. Use of non-programmable electronic calculator is allowed.
6. Figures to the right indicate full marks.

UNIT – I

1. a) i) Solve $\frac{d^2y}{dx^4} - y = \cos x \cosh x$ **4**

ii) Solve $x^3\frac{d^3y}{dx^3} + 2x^2\frac{d^2y}{dx^2} + 2y = 10\left(x + \frac{1}{x}\right)$ **4**

b) i) Solve by method of variation of parameter $\frac{d^2y}{dx^2} - y = \frac{2}{1+e^x}$. **4**

ii) Solve $(3x+2)^2\frac{d^2y}{dx^2} + 3(3x+2)\frac{dy}{dx} - 36y = 3x^2 + 4x + 1$ **4**

c) Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 2e^{2x} + 10\sin 3x$ given that $y(0) = 2,\ y'(0) = 4$. **8**

UNIT – II

2. a) A mass M suspended from the end of a helical spring is subjected to a periodic force $f = F\sin wt$ in the direction of its length. The force f is measured positive vertically downwords and at zero time M is at rest. It the spring stiffness is s, prove that the displacement of M at time t from the commencement of motion is given by $x = \frac{F}{M(p^2 - w^2)}\left(\sin wt - \frac{w}{p}\sin pt\right)$ where $p^2 = \frac{s}{M}$ and damping effects are neglected. **8**

b) Solve $\frac{\partial u}{\partial t} = K\frac{\partial^2 u}{\partial x^2}$ if **8**

i) $u(o,t) = 0 \quad \forall t$

ii) $u_x(\ell, t) = 0 \quad \forall t$

iii) $u(x,t)$ is bounded

iv) $u(x,o) = \frac{u_0 x}{\ell}, 0 < x < \ell$

c) Solve $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$ if **8**

i) $z(o,y) = 0$

ii) $z(x,\infty) = 0$

iii) $z(l,y) = 0$

iv) $z(x,o) = \sin^3 \pi x, 0 < x < 1$

UNIT – III

3. a) i) Find $L\left\{\int_0^t \frac{e^{-3t}\sin 2t}{t}\, dt\right\}$ **4**

ii) Find Laplace transform of $f(t) = \begin{cases} \sin t & 0 < t < \pi \\ \sin 2t & \pi < t < 2\pi \\ \sin 3t & t > 2\pi \end{cases}$ **4**

b) i) Evaluate $\int_0^\infty \frac{\cos 3t - \cos 2t}{t}\, dt$ **4**

ii) Find $L^{-1}\left\{\frac{6s-4}{s^2 - 4s + 20}\right\}$ **4**

c) i) Find $L^{-1}\left[\frac{s^2}{\left(s^2 + a^2\right)^2}\right]$ by convolution theorem. **4**

ii) Solve $y'' + y = t$ by using Laplace transform, given that y (0) = 1, y'(0) =- 2. **4**

UNIT – IV

4. a) Calculate the first four moments of the following distribution about the mean & hence find β_1 & β_2 and comment on distribution. **8**

x	0	1	2	3	4	5	6	7	8
f	1	8	28	56	70	56	28	8	1

b) i) In a normal distribution, 31% of the items are under 45 & 8% are over 64. Find the mean and standard deviation of the distribution. **4**

ii) Suppose 3% of bolts made by a machine are defective, the defects occuring at random during production. If bolts are packaged 50 per box. Find the Poisson approximation to it, that a given box will contain 5 defectives. **4**

c) Calculate the coefficient of correlation & obtain the regression line of y on x for the following data : **8**

x	1	2	3	4	5	6	7	8	9
y	9	8	10	12	11	13	14	16	15

Also obtain an estimate of y which should correspond on the average to x = 6.2.

UNIT – V

5. a) i) Find Fourier sine transform of the function **4**

$$f(x)=\begin{cases}\sin x & 0\le x\le a\\ 0 & x>a\end{cases}$$

And also find Fourier integral representation.

ii) Show that **4**

$\bar{F}=\left(y^2+2xz^2\right)\hat{i}+(2xy-z)\,\hat{j}+\left(2x^2z-y+2z\right)\hat{k}$

is irrotational and hence find its scalar potential.

b) i) Find unit normal vector to surface $\phi=3x^2y-y^3z^2$ at point (1, -2, 1). **4**

ii) Find the directional derivative of $\phi=e^{2x}\cos yz$ at the origin in the direction of the tangent to curve x = asint, y = acost, z = at at $t=\pi/4$. **4**

c) Find the Fourier sine transform of $\frac{e^{-ax}}{x}$ and hence evaluate **8**

i) $\int_0^\infty \tan^{-1}\frac{\lambda}{a}\sin\lambda x\ d\lambda$

ii) $\int_0^\infty \tan^{-1}\frac{\lambda}{a}\sin\lambda\ d\lambda$

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