Seat No.

* 2 7 3 3 *

Optimization Techniques

P. Pages : 3

Time : Three Hours

Max. Marks : 100

Instructions to Candidates :

1. Do not write anything on question paper except Seat No.
2. Answersheet should be written with blue ink only. Graph or diagram should be drawn with the same pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. Answer **any five** questions.
5. Neat diagram must be drawn wherever necessary.
6. Figures to the right indicate full marks.
7. Use of non programmable electronic pocket calculator is allowed.
8. Assume suitable data wherever necessary.

1. a) Find the maxima and minima, if any, of the function

$$f(x)=\frac{x^4}{(x-1)(x-3)^3}$$ 10

b) In a two-stage compressor, the working gas leaving the first stage of compression is cooled (by passing it through a heat exchanger) before it enters the second stage of compression to increase the efficiency. The total work input to a compressor (W) for an ideal gas, for isentropic compression, is given by

$$W = c_p T_1 \left[\left(\frac{p_2}{p_1}\right)^{(k-1)/k} + \left(\frac{p_3}{p_2}\right)^{(k-1)/k} - 2 \right]$$

where c_p is the specific heat of the gas at constant pressure, k is the ratio of specific heat at constant pressure to that at constant volume of the gas and T_1 is the temperature at which the gas enters the compressor. Find the pressure, p_2, at which intercooling should be done to minimize the work input to the compressor. Also determine the minimum work done on the compressor. 10

2. a) What are the limitations of classical methods in solving a one-dimensional minimization problem ? 5

b) Minimize :

$f(x) = 0.65 - \left[0.75/\left(1+x^2\right)\right] - 0.65x\tan^{-1}(1/x)$ using the interval halving method. 15

3. a) Solve the following LP problem in standard form :

Maximize $f = x_1 - 8x_2$

Subject to

$$3x_1 + 2x_2 \geq 6$$
$$9x_1 + 7x_2 \leq 108$$
$$2x_1 - 5x_2 \geq -35$$

x_1, x_2 unrestricted in sign. 10

b) Solve the following by simplex technique.

Maximize

$$z = 13x_1 + 14x_2 + 15x_3$$

Subject to

$-x_1 + x_2 \leq 0;\ -x_1 + 2x_3 \leq 0;\ x_1 + x_2 + x_3 \leq 150.$ 10

4. a) Using the box (simplex) method, carry one iteration for minimization of following function. Initial three points are (0, 0), (1, 1) and (2, 0). Also find the optimum length (λ) for the iteration.

$f(x_1, x_2) = 6x_1^2 + 2x_2^2 - 6x_1x_2 - x_1 - 2x_2$ 10

b) Minimize, by using Newton method.

$f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$, Starting point (0, 0). 10

5. Minimize $f(x_1, x_2) = 100\left(x_1^2 - x_2\right)^2 + (1 - x_1)^2$ taking $X_1 = \begin{Bmatrix} -2 \\ -2 \end{Bmatrix}$ as the starting point. 20

6. a) By using Lagrange multipliers, find the dimensions of a cylindrical tin (with top bottom) made up of steel metal to maximize its volume such that the total surface area is equal to $A_0 = 75.36$. 10

b) Solve the following system of equations using pivot operations : **10**

$$6x_1 - 2x_2 + 3x_3 = 11$$
$$4x_1 + 7x_2 + x_3 = 21$$
$$5x_1 + 8x_2 + 9x_3 = 48$$

7. Consider the following optimization problem :

Maximize, $f = x_1 - x_2$

subject to $x_1^2 + x_2 \geq 2$

$$4 \leq x_1 + 3x_2$$
$$x_1 + x_2^4 \leq 30$$

i) Find whether the design vector X = [1, 1] satisfies the Kuhn-Tucker conditions for a constrained optimum.

ii) What are the values of the Lagrange multipliers (λ) at the given design vector ? **20**

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