Seat No. | | | | | | |

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मास - 020

Optimization Techniques
(1060)

P. Pages : 2

Time : Three Hours

Max. Marks : 100

Instructions to Candidates :

1. Do not write anything on question paper except Seat No.
2. Answersheet should be written with blue ink only. Graph or diagram should be drawn with the same pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. Answer **any five** questions.
5. Neat diagrams must be drawn wherever necessary.
6. Figures to the right indicate full marks.
7. Use of non-programmable electronic pocket calculator is allowed.
8. Assume suitable data wherever necessary.

1. a) Find the maxima and minima, if any, of the function

$$f(x) = 4x^3 - 18x^2 + 27x - 7$$ **10**

b) A pipe of length l and diameter D has at one end a nozzle of diameter d through which water is discharged from a reservoir. The level of water in the reservoir is maintained at a constant value h above the center of nozzle. Find the diameter of the nozzle so that the kinetic energy of the jet is a maximum. The kinetic energy of the jet can be expressed as

$$\frac{1}{4}\pi\rho d^2\left(\frac{2gD^5h}{D^5+4fld^4}\right)^{3/2}$$

Where ρ is the density of water, f the friction coefficient and g the gravitational constant. **10**

2. a) Explain the algorithms of interval halving method. **5**

b) Minimize $f(x) = -\frac{0.5}{\sqrt{1+x^2}} + \sqrt{1+x^2}\left(1-\frac{0.5}{1+x^2}\right) - x$

Using Golden section method, take n = 6, interval (0, 3) **15**

3. a) Solve the following LP problem in standard form:

$$\begin{aligned} \text{Maximize} \quad & f = x_1 - 8x_2 \\ \text{Subject to} \quad & 3x_1 + 2x_2 \geq 6 \\ & 9x_1 + 7x_2 \leq 108 \\ & 2x_1 - 5x_2 \geq -35 \end{aligned}$$

x_1, x_2 unrestricted in sign **10**

b) Solve the following LP problem in standard form: **10**

$$\begin{aligned} \text{Maximize} \quad & f = -2x_1 - x_2 + 5x_3 \\ \text{Subject to} \quad & x_1 - 2x_2 + x_3 \leq 8 \\ & 3x_1 - 2x_2 \geq -18 \\ & 2x_1 + x_2 - 2x_3 \leq -4 \end{aligned}$$

4. a) Using the Powell method, carry two iteration for minimization of following function. Initial three points are (0, 0), (1, 1) and (2, 0). Also find the optimum length (λ) for the iteration. $f(x_1, x_2) = 6x_1^2 + 2x_2^2 - 6x_1x_2 - x_1 - 2x_2$ **10**

b) Minimize, by using steepest decent method

$f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$, Starting point (0, 0) **10**

5. Minimize $f(x_1, x_2) = 100(x_1^2 - x_2)^2 + (1 - x_1)^2$ taking

$x_1 = \begin{Bmatrix} -2 \\ -2 \end{Bmatrix}$ as the starting point. **20**

6. a) By using Lagrange multipliers, find the dimensions of a cylindrical tin (with top bottom) made up of steel metal to maximize its volume such that the total surface area is equal to $A_0 = 75.36$ **10**

b) **10**

$$\begin{aligned} \text{Minimize} \quad & f(x_1, x_2) = (x_1 - 1)^2 + (x_2 - 5)^2 \\ \text{Subject to} \quad & -x_1^2 + x_2 \leq 4 \\ & -(x_1 - 2)^2 + x_2 \leq 3 \end{aligned}$$

7. Solve the following optimization problem **20**

$$\begin{aligned} \text{Maximize} \quad & f = x_1 + 2x_2 + x_3 \\ \text{Subject to} \quad & 2x_1 + x_2 - x_3 \leq 2 \\ & -2x_1 + x_2 - 5x_3 \geq -6 \\ & 4x_1 + x_2 + x_3 \leq 6 \\ & x_i \geq 0, \; i = 1, 2, 3 \end{aligned}$$

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