Seat No.

* 1 8 4 9 *

Engineering Mathematics - II
(102113)

P. Pages : 4

Time : Three Hours | Max. Marks : 80

Instructions to Candidates :
1. Do not write anything on question paper except Seat No.
2. Answersheet should be written with blue ink only. Graph or diagram should be drawn with the same pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. Attempt **any two** subquestions in each unit.
5. Figures to the right indicate full marks.
6. Use of non-programmable calculator is allowed.

UNIT - I

1. Solve **any two.**

a) i) If $u = e^{x-at}\sin(x-at)$, prove that **4**

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$$

ii) Apply Euler's theorem to find **4**

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 y}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$$

where $u = \operatorname{cosec}^{-1} \frac{x^3 + y^3}{x^{1/2} + y^{1/2}}$

b) i) Considering $u \to x,\ y \to \theta, \phi$, find $\frac{\partial u}{\partial \theta} + i\frac{\partial u}{\partial \phi}$ **4**

where $x + y = 2e^{\theta}\cos\phi,\quad x - y = 2ie^{\theta}\sin\phi$

ii) Find $\frac{du}{dt}$, given $u = x^2 + y^2 + z^2$, where **4**

$x = e^{-t},\ y = e^{-t}\sin t$ and $z = e^{-t}\cos t$

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c) If $x = \dfrac{\cos\theta}{u}$, $y = \dfrac{\sin\theta}{u}$, then show that **8**

$$u\frac{\partial z}{\partial u} - \frac{\partial z}{\partial \theta} = (y - x)\frac{\partial z}{\partial x} - (y + x)\frac{\partial z}{\partial y}.$$

UNIT - II

2. Solve **any two.**

a) i) Verify that $JJ' = 1$, where $J = \dfrac{\partial(x,y)}{\partial(u,v)}$, **4**

and $x = u(1 - v)$, $y = uv$.

ii) Examine for the functional dependence between **4**

$u = \sin^{-1}x + \sin^{-1}y$, $v = x\sqrt{1 - y^2} + y\sqrt{1 - x^2}$
and find the relation if it exists.

b) i) The acceleration of a piston is calculated by

$f = \omega^2 r\left(\cos\theta + \dfrac{r}{\ell}\cos 2\theta\right)$. It is calculated for the values of

$\theta = 30^o$, $\dfrac{r}{\ell} = \dfrac{1}{u}$. If the values of θ, ω were each 1% small, then show that the calculated value of f is about 1.5% small. **4**

ii) If the kinetic energy $k = \dfrac{wv^2}{2g}$, find approximately the change in the kinetic energy as w changes from 49 to 49.5 and v changes from 1600 to 1590. Assume g = 9.8. **4**

c) A space probe in the shape of ellipsoid $4x^2 + y^2 + 4z^2 = 16$ enters the earth's atmosphere and its surface begins to heat. After one hour, the temperature at the point (x, y, z) on the surface of the probe is

$T(x, y, z) = 8x^2 + 4yz - 16z + 600$. Find the hottest points on the probe's surface by Lagrange's method. **8**

UNIT - III

3. Solve **any two.**

a) i) Trace the Cartesian curve $y^2 = \frac{x^2(a-x)}{a+x}$, by discussing symmetry, tangents, asymptote and region. **4**

ii) Obtain half - range sine series for the function **4**

$$f(x) = \begin{cases} x & \text{for} \quad 0 < x < \pi/2 \\ \pi - x & \text{for} \quad \frac{\pi}{2} < x < \pi \end{cases}$$

b) i) Trace the curve $r = a(1-\cos\theta)$. **4**

ii) Obtain half - range cosine series for the function $f(x) = (x-1)^2$, in the range $0 < x < 1$. **4**

c) Find the Fourier series to represent the function f(x) with period 2π given by **8**

$$f(x) = \begin{cases} x & \text{for} \quad 0 \le x \le \pi \\ 2\pi - x & \text{for} \quad \pi \le x \le 2\pi \end{cases}$$

Also deduce, $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots = \frac{\pi^2}{8}$.

UNIT - IV

4. a) i) Change the order of integration in the integral $\int_0^3 \int_1^{\sqrt{4-y}} f(x,y)\, dx\, dy$. **4**

ii) Evaluate $\int_0^a \int_0^{\sqrt{a^2-y^2}} \left(x^2+y^2\right) dx\, dy$ by changing into polar coordinates. **4**

b) Evaluate the triple integral **8**

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dx\, dy\, dz}{\sqrt{1-x^2-y^2-z^2}}$$

c) Find the whole area of the curve $a^2x^2 = y^3(2a - y)$ **8**

UNIT - V

5. a) i) Find by Taylor's series method the value of y at x = 0.1, given that

$$\frac{dy}{dx} = x^2y - 1, \quad y(0) = 1.$$ **4**

ii) Use Euler's modified method to find $y(0.1)$, $y(0.2)$ given that

$$\frac{dy}{dx} = yx^2 - 1.1y, \quad y(0) = 1.$$ **4**

b) Use Picard's method up to 2nd approximation to find an approximate value of y when x = 0.1, if $\frac{dy}{dx} = x - y^2$ and y = 1 at x = 0. **8**

c) Use Runge – Kutta's fourth order method to find y (1.1), y (1.2) given that

$$\frac{d^2y}{dx^2} = x^2 + y, \; y(1) = 1.5.$$ **8**

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