Seat No. | | | | | | |

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मक्षिका - 008

Engineering Mathematics - I
(101103)

P. Pages : 3

Time : Three Hours **Max. Marks : 80**

Instructions to Candidates :

1. Do not write anything on question paper except Seat No.
2. Answersheet should be written with blue ink only. Graph or diagram should be drawn with the same pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. Attempt **any two** sub questions from each unit.
5. All questions carry equal marks.
6. Figures to the right indicates full marks.
7. Use of non-programmable calculator is allowed.

UNIT - I

1. a) By considering ranks, solve the system of equations if consistent; **8**

$4x-2y+6z=8,\ x+y-3z=-1,\ 15x-3y+9z=21$

b) Find Eigen values and corresponding Eigen Vector's for the matrix. **8**

$$A=\begin{bmatrix}1 & 1 & -2\\ -1 & 2 & 1\\ 0 & 1 & -1\end{bmatrix}$$

c) i) Show that :

$$A=\frac{1}{3}\begin{bmatrix}1 & 2 & 2\\ 2 & 1 & -2\\ 2 & -2 & 1\end{bmatrix}$$ is orthogonal matrix. Hence find A^{-1} **4**

ii) Centre of arc of the circle is (2, 2, 2) and origin is (0,0,0), rotation is about X-axis through an angle of 45°. Find the centre of the arc of circle in new co-ordinate system. **4**

UNIT - II

2. a) If $x=\sin\theta,\ y=\sin 2\theta$ prove that

$$(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(n^2-4)y_n=0$$ **8**

b) If $y^{1/m} - y^{-1/m} = 2x$ prove that

$$y = 1 + mx + \frac{m^2x^2}{2!} + \frac{m(m^2-1^2)}{3!}x^3 + \frac{m^2(m^2-2^2)}{4!}x^4 + \ldots\ldots$$ **8**

c) i) Using Taylor's theorem find approximate value of $\sqrt{25.15}$. **4**

ii) Express $(x-2)^3 + 2(x-2)^2 + 4(x-2) + 5$ is powers of x. **4**

UNIT - III

3. a) Prove that $B(m, n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}} dx$ **8**

b) i) Evaluate $\int_0^\infty \sqrt[4]{x}\, e^{-\sqrt{x}}\, dx$ **4**

ii) Prove that

$$\int_0^\infty \frac{e^{-ax} - e^{-bx}}{x} dx = \log\left(\frac{b}{a}\right) \quad a > 0, b > 0$$ **4**

c) i) Evaluate

$$\int_0^t erf(4x)dx + \int_0^t erfc(4x)dx$$ **4**

ii) If $\phi(a) = \int_a^{a^2} \frac{\sin ax}{x} dx$, find $\frac{d\phi}{da}$. **4**

UNIT - IV

4. a) i) Solve : $\frac{dy}{dx} = \frac{y+1}{(y+2)e^y - x}$ **4**

ii) Solve : $\left(x^2 + y^2 + x\right)dx + xy\,dy = 0$ 4

b) i) Solve : $y^2 + \left(x - \frac{1}{y}\right)\frac{dy}{dx} = 0$ 4

ii) Solve : $\frac{dy}{dx} - y\tan x = y^4 \sec x$ 4

c) Find the current 'i' in the circuit having resistance 'R' and condensor of capacity 'c' in series with emf $E\sin\omega t$. 8

UNIT - V

5. a) i) Prove that :

$$\text{Sech}^{-1}(\sin\theta) = \log\cot\left(\frac{\theta}{2}\right)$$ 4

ii) Prove that :

$$\tanh\left(\log\sqrt{3}\right) = \frac{1}{2}$$ 4

b) i) Prove that $i\log\left(\frac{x-i}{x+i}\right) = 2\cot^{-1}(x)$ 4

ii) Prove that

$$\tan\left[i\log\left(\frac{a-ib}{a+ib}\right)\right] = \frac{2ab}{a^2 - b^2}$$ 4

c) i) Find tan hx if 5 sin hx - cos hx = 5. 4

ii) Find general and principle values of i^i. 4

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