Engineering Mathematics - I
(Old) (1040)

P. Pages : 4

Time : Three Hours **Max. Marks : 100**

Instructions to Candidates :
1. Do not write anything on question paper except Seat No.
2. Answer sheet should be written with blue ink only. Graph or diagram should be drawn with the same pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All the questions are compulsory.
5. Figures to the right indicate full marks.
6. Use of non-programmable calculator is allowed.

1. a) Attempt **any two.** **14**

a) Find two non-singular matrices P and Q such that PAQ is in normal form, where $A=\begin{bmatrix}1 & 2 & 3 & 4\\ 2 & 1 & 4 & 3\\ 3 & 0 & 5 & -10\end{bmatrix}$

b) Use matrix method to determine the values of λ for which the equations, $x+2y+z=3, x+y+z=\lambda, 3x+y+3z=\lambda^2$ are consistent and solve them for these values of λ.

c) Test whether the vectors (1, 2, 3,4), (0, 1, -1, 2), (1, 5, 1, 8) and (3, 7, 8, 14) are linearly dependent. If dependent find relation between them.

b) Attempt **any one.** **6**

p) State Cayley-Hamilton theorem and verify it for

$$A=\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}.$$

q) Find eigen values and eigen vectors for the matrix,

$$A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$$

2. a) Attempt **any two.** **14**

i) If, $x = \sin\theta, y = \sin 2\theta$ prove that

$$\left(1-x^2\right)y_{n+2} - (2n+1)xy_{n+1} - \left(n^2-4\right)y_n = 0.$$

ii) Show that, $e^{\theta} = 1 + \sin\theta + \frac{1}{2!}\sin^2\theta + \frac{2}{3!}\sin^3\theta + \frac{5}{4!}\sin^4\theta + ...$

iii) Expand $\log\sin x$ in powers of $(x-2)$.

b) Attempt **any two.** **6**

i) Find n^{th} derivative of $x^2 \sin x$.

ii) Evaluate $\lim\limits_{x \to 0} \log_x \tan_x$

iii) Evaluate $\lim\limits_{x \to 0} \left(\frac{2^x + 3^x + 4^x}{3}\right)^{1/x}$

3. Attempt **any four.** **20**

a) Evaluate $\int_0^1 (x \log x)^4 dx$

b) Evaluate $\int_0^{\pi/2} \sqrt{\tan\theta}\, d\theta \int_0^{\pi/2} \sqrt{\cot\theta}\, d\theta$

c) State and prove duplication formula for gamma function.

d) Show that error function of x is an odd function. Hence show that $\text{erfc}\,(x) + \text{erfc}\,(-x) = 2$.

e) Verify rule of differentiation under integral sign for

$$\int_a^{a^2} \log(ax)dx$$

f) Evaluate $\int_0^\infty \frac{e^{-\alpha x}\sin x}{x}dx$. Hence show that $\int_0^\infty \frac{\sin x}{x}dx = \frac{\pi}{2}$.

4. Solve **any four.** **20**

a) $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dx}{dy} = 1$

b) $\frac{dy}{dx} + \frac{y}{x}\log y = \frac{y}{x^2}(\log y)^2$

c) $\frac{dy}{dx} = \frac{2x+3y+4}{-3x+4y-5}$

d) $y(xy+1)dx + x\left(1+xy+x^2y^2\right)dy = 0$

e) $\left(\frac{x\,dx + y\,dy}{x\,dy - y\,dx}\right)^2 = \left(\frac{a^2 - x^2 - y^2}{x^2 + y^2}\right)$

f) $y\log y\,dx + (x - \log y)\,dy = 0$

5. a) Attempt **any two.** **14**

i) A particle is projected vertically upwards in the earth's gravitational field offering a resistance k (velocity)^2 per unit mass. If V_o is the initial velocity, prove that the greatest height attained is $\frac{1}{2k}\log\left(1 + \frac{kV_o^2}{g}\right)$.

ii) Find the current i in the circuit containing resistance R and condenser of capacity C in series with e.m.f. $E\sin\omega t$.

iii) If a thermometer is taken outdoors where the temperature is 0ºC, from a room at temperature 21ºC and The reading drops to 10ºC in 1 minute how long after the removal will the reading be 5ºC ?

b) Attempt **any one.** **6**

i) A brick wall for which k = 0.0015 is 25 cm thick. If the inner surface is at 20ºC and outer is at 0ºC, find the temperature in the wall as a function of the distance from outer surface and the heat lost per day through one square meter.

ii) The differential equation of the atmospheric pressure is $\frac{dp}{dh} = -g\rho$, where ρ is the density at a vertical height h above the ground and P is the atmospheric pressure. Assuming $P = k\rho$, where k is constant, prove that $P = p_0 \, e^{-gh/k}$, where p_0 is pressure at ground level.

****