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Engineering Mathematics - I
(101103)

P. Pages : 3

Time : Three Hours **Max. Marks : 80**

Instructions to Candidates :
1. Do not write anything on question paper except Seat No.
2. Answer sheet should be written with blue ink only. Graph or diagram should be drawn with the same pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. Attempt **any two** subquestion from each unit.
5. Diagrams should be given wherever necessary.
6. Use of logarithmic table, drawing instruments and non programmable calculators is permitted.
7. Figures to the right indicate full marks.

UNIT – I

1. a) For what values of λ and μ the system of equation **8**

$$2x+3y+5z=9$$
$$7x+3y-2z=8$$
$$2x+3y+\lambda z=\mu,$$

has i) no solution,
ii) unique solution,
iii) infinitely many solutions.

b) Find eigen values and eigen vectors for matrix A, **8**

$$A=\begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$$

c) **4**

i) Show that the matrix $A=\frac{1}{3}\begin{bmatrix} 2 & 2 & 1 \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{bmatrix}$ is orthogonal, and also find A^{-1}.

ii) By reflection about the line y=x, obtain the new vertices of triangle ABC, whose old vertices are A(5, 1), B(4, B) & C(3, 2). **4**

UNIT – II

2. a) If $\cos^{-1}\left(\frac{y}{b}\right)=\log\left(\frac{x}{n}\right)^{n}$, then prove that $x^2y_{n+2}+(2n+1)x\,y_{n+1}+2n^2y_n=0,$ and also find $y_n(0)$. **8**

b) Using Maclaurin's expansion, prove that ; **8**

$$e^x=1+\tan x+\frac{1}{2!}\tan^2 x-\frac{1}{3!}\tan^3 x-\frac{7}{4!}\tan^4 x-----$$

c) i) Expand $\tan^{-1}x$ in powers of (x-1) by Taylor's theorem. **4**

ii) Using Taylor's theorem find $\sin\left(30^{o}\,31'\right)$. **4**

UNIT – III

3. a) i) Evaluate $\int_0^{\infty}\frac{x^8\left(1-x^6\right)}{(1+x)^{24}}\,dx$ **4**

ii) Evaluate $\int_0^{\pi/2}\sqrt{\cot\theta}\;d\theta$ **4**

b) i) Evaluate $\int_0^{\infty}\frac{x^3}{3^x}\,dx$ **4**

ii) Evaluate $\int_0^{\infty}\frac{e^{-ax}-e^{-bx}}{x}\,dx$ **4**

c) i) Prove that $\int_a^b e^{-x^2}dx=\frac{\sqrt{\pi}}{2}\,[\mathrm{erf}(b)-\mathrm{erf}(a)],\,(b>a)$ **4**

ii) Prove that $\frac{d}{dt}\mathrm{erf}\left(\sqrt{t}\right)=\frac{e^{-t}}{\sqrt{\pi t}}$ **4**

UNIT – IV

4. a) Solve : **4**

i) $\left(4x^3y^2+y\cos xy\right)dx+\left(2x^4y+x\cos yx\right)dy=0.$

ii) $y(1+xy)dx + x(1+xy+x^2y^2)dy = 0.$ **4**

b) Solve : **4**

i) $\frac{dx}{dy} + \frac{1}{1+y^2}x = \frac{e^{-\tan^{-1}y}}{1+y^2}.$

ii) $\frac{dy}{dx} - y\tan x = y^4 \sec x.$ **4**

c) A pipe 20 cm in diameter contains steam at 150ºC and is protected with a covering 5 cm thick for which k=0.0025. If the temperature of the outer surface of the covering is 40ºC, find the temperature half way through the covering under steady state conditions. **8**

UNIT – V

5. a) i) If $\sin\left(\frac{\pi}{4}+ix\right) = \frac{1}{u+iv}$, where u, v are real then prove that $(u^2+v^2)^2 = 2(u^2-v^2).$ **4**

ii) If $u = \log\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$, then prove that $\tanh\frac{u}{2} = \tan\frac{\theta}{2}.$ **4**

b) i) Show that $\cos\left[i\log\frac{a+ib}{a-ib}\right] = \frac{a^2-b^2}{a^2+b^2}.$ **4**

ii) Express $\log[\sin(x+iy)]$ in the form (a+ib). **4**

c) i) If $\tan(A+iB) = x+iy$, then prove that $x^2+y^2+2x\cot 2A = 1$ and $x^2+y^2-2x\cot 2B = -1.$ **4**

ii) Prove that $\left(i^i\right)$ is wholly real and find its principal value, also show that the values of $\left(i^i\right)$ form a geometric progression. **4**

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