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Engineering Mathematics - II
(Old) (1140)

P. Pages : 4

Time : Three Hours **Max. Marks : 100**

Instructions to Candidates :
1. Do not write anything on question paper except Seat No.
2. Answer sheet should be written with blue ink only. Graph or diagram should be drawn with the same pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to the right indicates full marks.
6. Non-programmable electronic calculator is allowed.
7. Neat diagram must be drawn wherever necessary.

1. Attempt **any four.** **20**

a) If $u = \log\left(x^3 + y^3 + z^3 - 3xyz\right)$ prove that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x+y+z)^2}$

b) If u=f(r) where $r^2 = x^2 + y^2 + z^2$, prove that
$u_{xx} + u_{yy} + u_{zz} = f''(r) + \frac{2}{r} f'(r).$

c) If $u = \tan^{-1}\left(\frac{x^3 + y^3}{2x + 3y}\right)$ find the value of $x^2 u_{xx} + 2xy\, u_{xy} + y^2 u_{yy}$

d) If $x = e^u \operatorname{cosec} v,\ y = e^u \cot v$ & $z = e^{-2u} f(v)$ find the value of $xz_x + yz_y$.

e) If $x^2 = au + bv,\ y^2 = au - bv$ & $\phi = f(x,y)$ then prove that
$x\frac{\partial \phi}{\partial x} + y\frac{\partial \phi}{\partial y} = 2\left[u\frac{\partial \phi}{\partial u} + v\frac{\partial \phi}{\partial v}\right]$

f) A thermodynamics relation is given by f(p,v,t)=0 Evaluate
$\left(\frac{\partial p}{\partial v}\right)_t \left(\frac{\partial v}{\partial t}\right)_p \left(\frac{\partial t}{\partial p}\right)_v$

2. Attempt **any four.** **20**

a) Verify $JJ' = 1$ for $x = uv,\ y = \dfrac{u+v}{u-v}$.

b) If $u_1 = f(x_1), u_2 = f(x_1, x_2), u_3 = f(x_1, x_2, x_3), ---, u_n = f(x_1, x_2, --- x_n)$ then show that $\dfrac{\partial(u_1, u_2, ---, u_n)}{\partial(x_1, x_2, ---, x_n)} = \dfrac{\partial u_1}{\partial x_1} \cdot \dfrac{\partial u_2}{\partial x_2} \cdot \dfrac{\partial u_3}{\partial x_3} \cdot ---- \cdot \dfrac{\partial u_n}{\partial x_n}$, hence if x=cosu, y=sinu cosv, z=sinu sinv cosw, find $\dfrac{\partial(x, y, z)}{\partial(u, v, w)}$.

c) Test for functional dependence of $u = e^x \sin y, v = x + \log(\sin y)$, If so find the relation between them.

d) The resistance R of a circuit was found by using ohm's Law $I = \dfrac{E}{R}$ If there is an error of 0.1 ampere in reading of current I & 0.5 volts in volt E, find the corresponding possible percentage error in R, when I=15 ampere & E=100 volts.

e) Find the approximate value of $\left[(0.98)^2 + (1.94)^2 + (2.01)^2\right]^{1/2}$ by using partial differentiation.

f) A space probe in the shape of ellipsoid $4x^2 + y^2 + 4z^2 = 16$ enters in the earth's atmosphere & it surface begins to heat. After one hour the temp at point (x, y, z) on the surface of probe is $T(x, y, z) = 8x^2 + 4yz - 16z + 600$. Find hottest point on the probe's surface by using Lagrange's method.

3. a) Trace any one of the following curve, with justification. **6**

i) $a^2 y^2 = x^2 (a^2 - x^2)$. ii) $r = a(1 - \sin\theta)$.

b) Attempt **any two.** **14**

p) Prove that in $-\pi \le x \le \pi$

$$\cosh ax = \frac{2a}{\pi} \sinh a\pi \left[\frac{1}{2a^2} + \sum_{n=1}^{\infty} \frac{(-1)^n}{(n^2 + a^2)} \cos nx\right]$$

q) Find Fourier expansion of the function $f(x)=\begin{cases} e^{x}, & -1 \le x \le 0 \\ e^{-x}, & 0 \le x \le 1 \end{cases}$

r) In $0 \le x \le \pi$ prove that
$$x^2 = \frac{2}{\pi}\left[\left(\frac{\pi^2}{1}-\frac{4}{1^3}\right)\sin 1x + \left(\frac{-\pi^2}{2}\right)\sin 2x + \left(\frac{\pi^2}{3}-\frac{4}{3^3}\right)\sin zx + - - -\right]$$

4. A) Attempt **any two.** **14**

a) Evaluate $\int_0^\infty \int_0^\infty e^{-x^2\left(1+y^2\right)} x\,dx\,dy$

b) Evaluate $\iint_R \sqrt{\frac{a^2b^2-b^2x^2-a^2y^2}{a^2b^2+b^2x^2+a^2y^2}}\,dx\,dy$ over the area of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1.$

c) Evaluate $\iiint_V xyz\left(x^2+y^2+z^2\right)dx\,dy\,dz$ over the first octant of a sphere $x^2+y^2+z^2=a^2.$

B) Attempt **any one.** **6**

p) Find the total area of curve $r=a(1+\cos\theta)$.

q) Find the volume bounded by the paraboloid $x^2+y^2=az$ and cylinder $x^2+y^2=a^2$.

5. Attempt **any four.** **20**

a) If $\bar{r}$ is the position vector of a moving particle of mass 'm' w.r.t. origin 'O' & $\bar{F}$ is the external force on the particle, then show that the moment of $\bar{F}$ about 0 is given by $\bar{M}=\frac{d\bar{H}}{dt}$, where $\bar{H}=\bar{r}\times m\bar{V}$, where $\bar{V}$ is velocity vector.

b) Find the directional derivative of $\phi=e^{2x-y-z}$ at $(1,1,1)$ in the direction of the tangent to the curve $x=e^t\sin t,\ y=e^t\cos t,\ z=e^t$ at t=0.

c) Find the constant a, b & c if $\overline{F} = \left(axye^{z}\right)i - \left(bx^2e^{z}\right)j + \left(cx^2ye^{z}\right)k$ is irrotational, hence find the corresponding scalar potential, using values of a, b & c.

d) If a particle describe a curve r = a cosecθ with constant angular velocity w, find the radial & transverse component of velocity & acceleration.

e) Find the angle of intersection between two surfaces $x\log z + 1 - y^2 = 0$ & $x^2y + z = 2$ at point (1,1,1).

f) Show that the vector field $\overline{F} = f(r)\bar{r}$ is always irrotational, and determine f(r) such that the field is solenoidal.

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