Engineering Mathematics - II
(102113)

P. Pages : 3

Time : Three Hours **Max. Marks : 80**

Instructions to Candidates :
1. Do not write anything on question paper except Seat No.
2. Answer sheet should be written with blue ink only. Graph or diagram should be drawn with the same pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. Attempt **any two** sub question from each unit.
5. Diagram should be given wherever necessary.
6. Use of logarithmic table, drawing instruments and non programmable calculators is permitted.
7. Figures to the right indicate full marks.

UNIT – I

1. a) i) If $u = \log(\tan x + \tan y + \tan z)$, then prove that **4**

$$\sin 2x \frac{\partial u}{\partial x} + \sin 2y \frac{\partial u}{\partial y} + \sin 2z \frac{\partial u}{\partial z} = 2$$

ii) If $u = \sin^{-1}\left(\sqrt{x^2 + y^2}\right)$, then using Euler's theorem, find the value of **4**

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}.$$

b) i) If $u = f(y - z, z - x, x - y)$, then find the value of $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$. **4**

ii) If $(\tan x)^y + y^{\cot x} = 5$, then find $\frac{dy}{dx}$. **4**

c) If $x = \sqrt{vw}$, $y = \sqrt{uw}$, $z = \sqrt{uv}$ and ϕ is a function of x, y, z then prove that **8**

$$x\frac{\partial \phi}{\partial x} + y\frac{\partial \phi}{\partial y} + z\frac{\partial \phi}{\partial z} = u\frac{\partial \phi}{\partial u} + v\frac{\partial \phi}{\partial v} + w\frac{\partial \phi}{\partial w}.$$

UNIT – II

2. a) i) If $x = u(1-v)$, $y = uv$, then verify $JJ' = 1$. **4**

ii) If $xu = vw$, $yv = wu$, $zw = uv$, then find $\dfrac{\partial(x, y, z)}{\partial(u, v, w)}$. **4**

b) i) If the H.P. required to proper a steamer varies as the cube of velocity (v) and square of the length (ℓ). If there is 2% increase in velocity and 3% increase in length, then find the % increase in H.P. **4**

ii) If $f(x, y) = \tan^{-1}(xy)$, then find $f(0.9, 1.2)$ approximately. **4**

c) Using Lagrange's method of undetermined multiplier, show that the stationary value of $u = x^m y^n z^p$ under the condition $x + y + z = a$ is $m^m n^n p^p \left(\dfrac{a}{(m+n+p)}\right)^{m+n+p}$. **8**

UNIT – III

3. a) i) Trace the curve $y^2 = \dfrac{x^2(a-x)}{(a+x)}$. **4**

ii) Find Fourier series expansion for the function $f(x) = x$, in $(-\pi, \pi)$. **4**

b) i) Trace the curve $r = a(1+\cos\theta)$. **4**

ii) Find half-range sine series for $f(x) = \ell x - x^2$ in $(0, \ell)$. **4**

c) Obtain Fourier series expansion for $f(x) = \left(\dfrac{\pi - x}{2}\right)^2$ in $(0, 2\pi)$ and $f(x) = f(x + 2\pi)$, and hence deduce $\dfrac{1}{1^2} + \dfrac{1}{2^2} + \dfrac{1}{3^2} + \ldots\ldots = \dfrac{\pi^2}{6}$. **8**

UNIT - IV

4. a) i) Evaluate $\iint\limits_R y\,dx\,dy$ where R is region bounded by $y^2 = 4x$ and $x^2 = 4y$. **4**

ii) Change the order of integration and evaluate $\int_0^a \int_y^a dx\,dy$. **4**

b) Evaluate $\iiint x^{a-1} y^{b-1} z^{c-1} dx\,dy\,dz$, taken throught the volume of tetrahedron given by $x \geq 0,\ y \geq 0,\ z \geq 0$ and $x + y + z \leq 1$. **8**

c) Find the volume bounded by cylinder $x^2 + y^2 = 4$ and the planes $y + z = 4$ and $z = 0$. **8**

UNIT – V

5. a) i) From the Taylor series for y(x), find y(0.3) correct to four decimal places if y(x) satisfies $\frac{dy}{dx} = x^2 y - 1$ and $y(0) = 1$. **4**

ii) Use Euler's modified method to find approximate value of y, satisfying the differential equation $\frac{dy}{dx} = \log(x + y)$, given y(1) = 2, for x = 1.2 correct to three decimal places; h = 0.2. **4**

b) i) Use Picard's method to obtain 'y' at $x = D \cdot 2$, correct to three decimal places, for the equation $\frac{dy}{dx} = x - y$, with the condition y = 1, when x = 0. **4**

ii) Given $\frac{dy}{dx} = x^2 - y,\ y(0) = 1$ find y at x = 0.2 taking h = 0.1 using second order Runge – Kutta method. **4**

c) Using fourth order Runge-Kutta method, find approximate value of y at x = 0.2 taking h = 0.1, given that $\frac{dy}{dx} = \sqrt{x + y}$ with $x = 0,\ y = 1$. **8**

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