Seat No. | | | | | | |

**AOI 1325**

# Engineering Mathematics-II (Old) (1140)

**P. Pages: 3** **Max. Marks: 100**

**Time:** Three Hours

**Instructions to Candidates:**

1. Do not write anything on question paper except seat number.
2. Answer sheet should be written with blue ink only. Graph or diagram should be drawn with the same pen being used for writing paper or black HB pencil.
3. Student should note, no supplement will be provided.
4. Figures to the right indicate full marks.
5. All questions are compulsory.
6. Use of non-programmable electronic calculator is allowed.

**1** **Attempt any Four** **20**

a If $w = \log(2x + 2y) + \tan(2x - 2y)$, find the value of $\left(\frac{\partial^2 w}{\partial x^2} - \frac{\partial^2 w}{\partial y^2}\right)$.

b If $x = r\cos\theta, y = r\sin\theta$ then find the value of $i)\left(\frac{\partial\theta}{\partial x}\right)_y$ $ii)\left(\frac{\partial\theta}{\partial y}\right)_x$ $iii)\left(\frac{\partial r}{\partial\theta}\right)_x$ $iv)\left(\frac{\partial\theta}{\partial r}\right)_y$.

c Verify Euler's theorem for the function $u = (ax + by)^{\frac{1}{3}}$.

d Show that $x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} = \frac{\tan u}{12}\left(\frac{13}{12} + \frac{tan^2 u}{12}\right)$ when $(\sqrt{x} + \sqrt{y})sin^2u = x^{\frac{1}{3}} + y^{\frac{1}{3}}$.

e Find $\frac{dy}{dx}$ from the given implicit function $f(x,y) = \sin xy - e^{xy} - x^2y = 0$.

f If $u = f(y - z, z - x, x - y)$ then find the value of $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$

**2** **Attempt any Four** **20**

a Calculate $\frac{\partial(x,y,z)}{\partial(u,v,w)}$ if $u = \frac{2yz}{x}, v = \frac{3zx}{y}, w = \frac{4xy}{z}$

AOI 1325

**b** Given $x = u + v + w, y = u^2 + v^2 + w^2, z = u^3 + v^3 + w^3$ show that $\frac{\partial u}{\partial x} = \frac{vw}{(u-v)(u-w)}$

**c** Find whether the following functions are functionally dependent or not. If so, find the relation between them $u = \sin x + \sin y, v = \sin(x + y)$.

**d** Find the possible percentage error in computing the parallel resistance $r$ of two resistances $r_1$ and $r_2$ from the formula $\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2}$ where $r_1$ and $r_2$ are both in error by +3% each.

**e** Find approximate value of $\sqrt{(1.02)^2 + (3.98)^2 + (2.01)^2}$ by using theory of approximation.

**f** Divide 24 into three parts such that the continued product of the first, square of the second and the cube of the third may be maximum. Use Lagrange's method of multiplier.

**3** **Attempt any One** **6**

**a** i) Trace the curve $r = a \cos 2\theta$ with justification.

ii) Trace the curve $x(x^2 + y^2) = a(x^2 - y^2), a > 0$ with justification.

**b** **Attempt any Two** **14**

i) Expand $f(x) = x^2, 0 < x < 4$ with period=4 in a Fourier series.

ii) Obtain the Fourier series for the following function with period=$2\pi$.

$$f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$$

iii) Expand

$$f(x) = \begin{cases} \frac{1}{4} - x, & 0 < x < \frac{1}{2} \\ x - \frac{3}{4}, & \frac{1}{2} < x < 1 \end{cases}$$

as a half range sine series.

**4** **Attempt any Two** **14**

**a** i) Evaluate $\iint_R e^{2x+3y} dxdy$ over the triangle bounded by $x = 0, y = 0$ and $x + y = 1$.

ii) Change the order of the integration and hence evaluate $\int_0^1 \int_x^{2-x} \frac{x}{y} dydx$

iii) Evaluate $\iiint \frac{dxdydz}{(x+y+z+1)^3}$ if the region of integration is bounded by the co-ordinate planes and the plane $x+y+z=1$. 6

b Attempt Any One

i) Find the area of $r = a(1+\cos\theta)$, using double integration.

ii) Find the volume of sphere $x^2+y^2+z^2=a^2$, using triple integration.

5 **Attempt any Four** 20

a Find the angle between the tangents to the curve $\bar{r} = t^2 i + 2tj - t^3 k$ at the points $t = \pm 1$.

b Find the directional derivative of the scalar function $f(x,y,z) = x^2 + xy + z^2$ at the point A(1,-1,-1) in the direction of the line AB where B has co-ordinates (3,2,1).

c A particle moves along the curve $x = \cos t, y = \sin t, z = t$ with constant angular velocity $\omega$. Find the radial and transverse components of its linear velocity and acceleration at any time t.

d Find the directional derivatives of $f(x,y,z) = xyi + xy^2 j + z^2 k$ at the point (2,1,2) in the direction of the outer normal to the sphere $x^2+y^2+z^2=9$.

e A fluid motion is given by $v = (y+z)i + (z+x)j + (x+y)k$. Is this motion irrotational? If so, find the velocity potential.

f A particle describes the curve $r^2 = a^2\cos 2\theta$. Under the action of a force directed towards pole. Find the law of force.