Seat No. | | | | | | |

2509

AOI1301

Engineering Mathematics - I
(101103)

P. Pages : 4

Time : Three Hours

Max. Marks : 80

Instructions to Candidates :
1. Do not write anything on question paper except Seat No.
2. Answersheet should be written with blue ink only. Graph or diagram should be drawn with the same pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. Attempt **any two** sub questions from each unit.
5. Use of non programable calculator is permitted.
6. Figures to the right indicate full marks.
7. Assume suitable data wherever necessary.

UNIT - I

1. a) By considering rank of relevant matrices, examine for consistency if so find solution. **8**

$$x_1 + x_2 + x_3 - x_4 = -2$$
$$3x_1 + 2x_2 - x_3 = 6$$
$$2x_1 + x_2 - x_3 + 3x_4 = 8$$
$$4x_2 + 3x_3 + 2x_4 = -8$$

b) Find the eigen values and corrosponding eigen vectors of the matrix. **8**

$$A = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$$

c) i) If $A = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} & a \\ \frac{2}{3} & \frac{1}{3} & b \\ \frac{2}{3} & \frac{-2}{3} & c \end{bmatrix}$ is orthogonal matrix. Find the values of a, b, c. **4**

ii) Centre of arc of the circle is (10, 10, 10) origin (0, 0, 0) rotation is about x axis through an angle 60° Find centre of arc of the circle in new co-ordinate system. **4**

UNIT - II

2. a) If $y = \left[\log\left(x + \sqrt{x^2+1}\right)\right]^2$ show that $(y_{n+2})_0 = -n^2(y_n)_0$ **8**

b) Expand $\log\left[1+e^x\right]$ by Maclaurin's theorem up to x^4. **8**

c) i) Expand $2x^3 + 7x^2 + x - 6$ in powers of $(x-2)$. **4**

ii) Use Taylor's theorem to find $\sqrt{25.15}$. **4**

UNIT - III

3. a) Evaluate $\int_0^1 \left[\frac{x^2+x^3}{(1+x)^7}\right] dx$. **8**

b) i) Evaluate $\int_0^\infty \sqrt[4]{x} \cdot e^{-\sqrt{x}}\, dx$. **4**

ii) Evaluate $\int_0^\infty \frac{e^{-\alpha x}\sin x}{x} dx$. **4**

c) i) Prove that $\int_a^b e^{-x^2} dx = \frac{\sqrt{\pi}}{2}[\mathrm{erf}(b) - \mathrm{erf}(a)]$. $b>a$ **4**

ii) Prove that $\int_0^\infty e^{-x^2-2bx} dx = \frac{\sqrt{\pi}}{2} e^{b^2}[1-\mathrm{erf}(b)]$. **4**

 AOI1301

UNIT - IV

4. a) Solve

i) $\frac{dy}{dx} + \frac{y\cos x + \sin y + y}{\sin x + x\cos y + x} = 0$ **4**

ii) $y(1+xy)dx + x(1+xy+x^2y^2)dy = 0$. **4**

b) Solve:

i) $(x+2y^3)\frac{dy}{dx} = y$ **4**

ii) $\frac{dy}{dx} + xy = y^2 e^{\frac{x^2}{2}} \log x$. **4**

c) A voltage Ee^{-at} is applied at t = 0 to a circuit containing inductance L, aresistance R show that the current at any time 't' is

$$\frac{E}{R-aL}\left[e^{-at} - e^{\frac{-R}{L}t}\right].$$ **8**

UNIT - V

5. a) i) If $\cosh x = \sec\theta$ then show that $\theta = \frac{\pi}{2} - 2\tan^{-1}\left[e^{-x}\right]$. **4**

ii) If $\tan(\alpha + i\beta) = x + iy$ prove that $x^2 + y^2 + 2x\cot 2\alpha = 1$. **4**

b) i) If $\tan[\log(x+iy)] = a + ib$ show that

$\tan\left[\log(x^2+y^2)\right] = \frac{2a}{1-a^2-b^2}$ where $(a^2+b^2 \neq 1)$. **4**

AOI1301

ii) Prove that $\log_{(1-i)}^{(1+i)} = \dfrac{\frac{1}{4}(\log^2)^2 - \frac{\pi^2}{16} + i\frac{\pi}{4}\log^2}{\frac{1}{4}(\log^2)^2 + \frac{\pi^2}{16}}$. **4**

c) i) Prove that i^{2i} is wholly real and find its principal value. **4**

ii) Seprate in to real and imaginary part of $\cos^{-1}\left[\frac{3i}{4}\right]$. **4**

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