Seat No.

* 2 5 0 2 *

Engineering Mathematics - I
(Old) (1040)

P. Pages : 4

Time : Three Hours | Max. Marks : 100

Instructions to Candidates :
1. Do not write anything on question paper except Seat No.
2. Answersheet should be written with blue ink only. Graph or diagram should be drawn with the same pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to the right indicates full marks.
6. Non-programmable electronic calculator is allowed.
7. Assume suitable data if necessary.

UNIT - I

1. a) Attempt **any two.** **14**

i) Test the consistency and solve if consistent.

$$x+2y-z=3$$
$$3x-y+2z=1$$
$$2x-2y+3z=2$$
$$x-y+z=-1$$

ii) Find the Eigen values and corresponding eigen vectors of the matrix.

$$A=\begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$$

iii) Test whether the vectors are linearly dependent or not. If dependent find the relation between them.

$x_1=(1, 2, -1, 0)$, $x_2=(1, 3, 1, 2)$, $x_3=(4, 2, 1, 0)$, $x_4=(6, 1, 0, 1)$.

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b) Attempt **any one.** **6**

i) Verify Cayley-Hamilton theorem for the matrix.

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}.$$

ii) Find ℓ, m, n so that the matrix is orthogonal where

$$A = \begin{bmatrix} o & 2m & n \\ \ell & m & -n \\ \ell & -m & n \end{bmatrix}.$$

UNIT - II

2. a) Attempt **any two.** **14**

i) If $y = e^{m\cos^{-1}x}$ prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0$.

ii) Show that $\sin^{-1}x = x + \frac{1^2}{3!}x^3 + \frac{1^2 \cdot 3^2}{5!}x^5 + \frac{1^2 \cdot 3^2 \cdot 5^2}{7!}x^7 + \ldots\ldots\ldots$

iii) Using Taylor's theorem find the expansion of $\tan\left(x + \frac{\pi}{4}\right)$ up to x^4 and hence find the value of tan(42°).

b) Attempt **any one.** **6**

i) Evaluate $\lim\limits_{x\to 0} \tan x \log x$.

ii) Evaluate $\lim\limits_{x\to 0} \dfrac{e^x + \log\left(\frac{1-x}{e}\right)}{\tan x - x}$.

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UNIT - III

3. a) Attempt **any two.** **14**

i) Evaluate using Gamma function $\int_0^\infty \frac{e^{-x^3}}{\sqrt{x}} dx \times \int_0^\infty \frac{x^4}{4^x} dx$.

ii) Prove that $\beta(m,n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}} dx$.

iii) Prove that $\int_0^\infty \frac{\cos \lambda x}{x}\left(e^{-ax} - e^{-bx}\right)dx = \frac{1}{2}\log\left(\frac{b^2+\lambda^2}{a^2+\lambda^2}\right)$ $a > 0,\ b > 0$.

b) Attempt **any one.** **6**

i) Evaluate using Gamma function $\int_0^1 (\log x)^5 \, dx$.

ii) Define error function, complimentary error function and show that $\operatorname{erfc}(x) + \operatorname{erfc}(-x) = 2$.

UNIT - IV

4. Attempt **any four.** **20**

a) Solve $(xdy - ydx) = (x^2 + y^2)\,(xdx + ydy)$.

b) Solve $\frac{dy}{dx} + x\tan(y - x) = 1$.

c) Solve $y(xy+1)dx + x(1 + xy + x^2y^2)dy = 0$.

d) Solve $\cos y - x \sin y \frac{dy}{dx} = \sec^2 x$.

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e) Solve $ye^y = (y^3 + 2xe^y)\frac{dy}{dx}$.

f) Solve $\frac{dy}{dx} - y\tan x = y^4 \sec x$.

UNIT - V

5. a) Attempt **any two.** **14**

i) The equation of L - R circuit is given by $L\frac{di}{dt} + Ri = 10\sin t$. If i = 0 at t = 0, express 'i' as a function at 't'.

ii) A body originally at 80°C cools down to 60°C in 20 minutes. the tempreature of the air being 40°C. What will be the tempreature of the body after 40 minutes from the original.

iii) A body at mass 'm' falls from rest under gravity in a fluid whose resistance to motion at any instant is 'mk' times velocity, where 'k' is constant. Find the terminal velocity of the body and also time taken to aquire one half of its limiting speed.

b) Attempt **any one.** **6**

i) The differential equation of the atmospheric pressure is

$\frac{dp}{dh} = -g\rho$ where ρ is the density at a vertical height 'h' above the ground and 'p' is atmospheric pressure. Assuming $p = k\rho$, where k is constant.

Prove that $P = P_o e^{-gh/k}$ where P_o is the pressure at the ground level.

ii) A long hollow pipe has an inner diameter of 10 cm. and outer diameter of 26cm. The inner surface is kept at 210°C and the outer surface at 50°C. If the thermal conductivity is 0.12. Find the tempreature at a distance x = 9cm. from the center of pipe.

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