



## Engineering Mathematics - III (1050)

P. Pages : 4

Time : Three Hours

Max. Marks : 100

Instructions to Candidates :

1. Do not write anything on question paper except Seat No.
2. Answer sheet should be written with blue ink only. Graph or diagram should be drawn with the same pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to right indicate full marks.
6. Use of statistical data is allowed.
7. Use of non-programmable calculator is allowed.

1. Attempt **any four**.

20

a) Solve :  $\frac{d^2y}{dx^2} + y = x \cos 2x + \frac{3}{2}$

b) Solve :  $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 4x^2e^x + 2^x$

c) Solve  $\frac{d^2y}{dx^2} + 4y = \tan 2x$  by method of variation of parameters.

d) Solve :  $x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + 4y = \cos(\log x) + x \sin(\log x)$

e) Solve :  $\frac{du}{dx} + v = \sin x$   
 $\frac{dv}{dx} + u = \cos x$  given that

when  $x = 0$ ,  $u = 1$  and  $v = 0$ .f) An e.m.f.  $E \sin pt$  is applied at  $t = 0$  to a circuit containing a condenser  $C$  and inductance  $L$  in series. The current  $x$  satisfies the equation.

$L \frac{dx}{dt} + \frac{1}{C} \int x dt = E \sin pt$ , where  $x = -\frac{dq}{dt}$  If  $P^2 = \frac{1}{LC}$  and initially the current  $x$  and the charge  $q$  are zero then show that the current in the circuit at time  $t$  is given by,  $\frac{E}{2L} t \sin pt$ .

2. a) Attempt **any two**.

12

- i) Find the Fourier integral representation of the function

$$f(x) = \begin{cases} 1; & |x| < 1 \\ 0; & |x| > 1 \end{cases} \text{ and hence evaluate (p) } \int_0^{\infty} \frac{\sin \lambda \cos \lambda x}{\lambda} d\lambda$$

$$(q) \text{ deduce the value of } \int_0^{\infty} \frac{\sin \lambda}{\lambda} d\lambda.$$

- ii) Solve the following difference equation using z-transform

$$f(k+1) + \frac{1}{3}f(k) = \left(\frac{1}{3}\right)^k, k \geq 0, f(0) = 0$$

- iii) Using Fourier integral representation, show that

$$\int_0^{\infty} \frac{\lambda^3 \sin \lambda x}{\lambda^4 + 4} d\lambda = \frac{\pi}{2} e^{-x} \cos x, \text{ where } x > 0$$

b) Attempt **any two**.

8

- i) Find
- $z\{f(k)\}$
- if
- $f(k) = \frac{\sin ak}{k}, k > 0$

- ii) Find
- $z^{-1} \left\{ \frac{z^2}{(z - \frac{1}{2})(z - \frac{1}{3})} \right\}$
- if
- $\frac{1}{3} < |z| < \frac{1}{2}$

- iii) Find the Fourier sine transform of
- $\frac{1}{x}$
- .

3. Attempt **any four**.

20

- a) Find
- $L \left\{ \int_0^t t e^{-3t} \cos 4t dt \right\}$

- b) Evaluate
- $L \int_0^{\infty} e^{-t} \frac{(1 - \cos t)}{t} dt$

- c) Find
- $L \{ t^2 U(t-4) + \sin(t-\pi) U(t-\pi) + t^3 \delta(t-2) \}$

- d) Find
- $L^{-1} \left\{ \tan^{-1} \left( \frac{S+3}{5} \right) \right\}$

- e) Find
- $L^{-1} \left\{ \frac{S}{(S^2 + 4)^2} \right\}$
- using the convolution theorem.

- f) Solve using Laplace transform.

$$y'' + y = 0, \text{ given } y(0) = 1, y'(0) = 2.$$

4. Attempt **any two**.

- a) i) Given the distribution of wages in two factories x and y.

6

Wages in Rs.	No. of workers x	No. of workers y
50 - 100	2	6
100 - 150	9	11
150 - 200	29	18
200 - 250	54	32
250 - 300	11	27
300 - 350	5	11

State in which factory the wages are more variable.

- ii) The first four moments of a distribution about
- $x = 2$
- are 1, 2.5, 5.5 and 16. Calculate the first four moments about the mean.

4

- b) i) Find the coefficient of correlation for the following data :

6

x	10	14	18	22	26	30
y	18	12	24	6	30	36

- ii) The overall percentage of failures in a certain examination is 20. If six candidates appear in the examination, what is the probability that at least five pass the examination?

4

- c) i) Find the lines of regression for the following data :

6

x	1	3	4	6	8	9	11	14
y	1	2	4	4	5	7	8	9

Estimate y for  $x = 10$ 

- ii) A manufacturer knows that the condensers he makes contain on an average 1% of defectives. He packs them in boxes of 100. What is the probability that a box picked at random will contain 4 or more faulty condensers?

4

5. Attempt **any two**.

- a) i) A sample of 256 bricks has mean weight of 2.12kg. with standard deviation of 560 gm. Test the hypothesis that the sample come from a population with mean weight of 2kg. **5**
- ii) State Beta and Gamma distribution with their mean and variance. **5**
- b) i) A Certain Factory runs in two shifts. A sample of 1000 items selected from production of day shift, gave 52 defective articles. However a sample of 700 items selected from production of night shift revealed 45 items defective. Can we conclude that proportion of defective items in the first shift is less than that of second shift? **5**
- ii) Write a short note on chi-square distribution. **5**
- c) i) A company producing spark plugs claimed that there would be 10% defective spark plugs. When a sample of 500 was taken 62 were found defective. Test correctness of company's claim. **5**
- ii) Fit a Poisson distribution for the following data and test the goodness of fit.  $\left[\chi^2_{2,0.05} = 5.991\right]$ . **5**

x	0	1	2	3	4	5	6
F	273	70	30	7	7	2	1

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